# Math 114
# Functions, Exponentials & Logarithms Review

## You should be able to solve the following without a calculator

[0]    Complete the following definitions.

   [a]    $y$ is a function of $x$ if and only if     _____
   [b]    $y$ is a one-to-one function of $x$ if and only if     _____
   [c]    The domain of a function is     _____
   [d]    The range of a function is     _____

[1]    Find the domains of the following functions.

   [a]    $f(x) = 4^x$          [b]    $f(x) = \log_3 x$          [c]    $f(x) = \log_5(12 - 6x)$

[2]    Find the ranges of the following functions.

   [a]    $f(x) = 5^x$          [b]    $f(x) = \log_7 x$

[3]    Evaluate the following. Write "UNDEFINED" if the value does not exist.

   [a]    $\log_3 81$          [b]    $\log_6 36$          [c]    $\log_5 125$          [d]    $\log_7 0$
   [e]    $\log_8 -8$          [f]    $\log_6 1$          [g]    $\log_2 64$          [h]    $\log 10000$
   [i]    $\log_4 4^6$          [j]    $\log_8 8^{-3}$          [k]    $3^{\log_3 7}$          [l]    $6^{\log_6 0}$
   [m]    $5^{\log_5 -10}$          [n]    $10^{\log 5}$

[4]    Find the exact solutions of the following equations. **Check your answers.**

   [a]    $3^{2-x} = 81$                                [b]    $8^{3x-7} = 4^{6-x}$
   [c]    $1 + 2\log_4(5x + 9) = 7$                     [d]    $\log_3(x^2 - 7) - \log_3(1 - x) = 1$
   [e]    $\log_2(10x - 2) - \log_2(x + 1) = 3$          [f]    $\log(2x + 6) + \log(x - 2) = 2$

[5]    Write as the logarithm of a single quantity. Simplify your answer.

   [a]    $\log 8 + \log 6 - \log 2$                     [b]    $\log 48 - \log 6 - \log 2$
   [c]    $5\log 2$                                      [d]    $3\log x + 2\log y$
   [e]    $2\log x - \log y + \log z$                    [f]    $\log z - 2\log y - \log x$
   [g]    $2\log y + 3\log z - \log x$                   [h]    $4\log z + \log x + 3\log y$

[6]    Write as the sums and/or differences and/or multiples of logarithms of numbers or single variables.

   [a]    $\log(7 \times 11)$          [b]    $\log\left(\dfrac{13}{5}\right)$          [c]    $\log 3^8$          [d]    $\log r^4 s$

   [e]    $\log\dfrac{a^5}{b^2}$          [f]    $\log\dfrac{m}{n^2 p^3}$          [g]    $\log\dfrac{x^2}{\sqrt{yz}}$

[7]    Find the domains of the following functions.

   [a]    $f(x) = x^2 + 3x$                              [b]    $f(x) = \dfrac{5}{2x - 3} - 1$

   [c]    $f(x) = \sqrt{8 - x} - 6$

[8]     Find the ranges of the following functions.

[a]     $f(x) = \dfrac{2}{x+5} - 4$

[b]     $f(x) = 7 - \sqrt{x+9}$

[9]     **MULTIPLE CHOICE**

[a]     The graph of $f(x) = 3\log(x+4)$ has an asymptote at
        [i]     $x = 4$          [ii]     $x = -4$          [iii]     $y = -4$          [iv]     $y = 4$          [v]     $y = 3$

[b]     The graph of $f(x) = -5 \cdot 2^{x-3}$ has an asymptote at
        [i]     $x = 3$          [ii]     $x = 0$          [iii]     $y = -5$          [iv]     $y = 3$          [v]     $y = 0$

[c]     For the logarithm curve $f(x) = \log_2 x$, as the value of $x \to \infty$, the value of $y \to$
        [i]     $\infty$          [ii]     $-\infty$          [iii]     $0$          [iv]     $1$          [v]     $-1$

[d]     For the logarithm curve $f(x) = \log_5 x$, as the value of $x \to 0$, the value of $y \to$
        [i]     $\infty$          [ii]     $-\infty$          [iii]     $0$          [iv]     $1$          [v]     $-1$

[e]     For the exponential curve $f(x) = \left(\dfrac{5}{3}\right)^x$, as the value of $x \to \infty$, the value of $y \to$
        [i]     $\infty$          [ii]     $-\infty$          [iii]     $0$          [iv]     $1$          [v]     $-1$

[f]     For the exponential curve $f(x) = \left(\dfrac{5}{3}\right)^x$, as the value of $x \to -\infty$, the value of $y \to$
        [i]     $\infty$          [ii]     $-\infty$          [iii]     $0$          [iv]     $1$          [v]     $-1$

[g]     For the exponential curve $f(x) = \left(\dfrac{5}{7}\right)^x$, as the value of $x \to \infty$, the value of $y \to$
        [i]     $\infty$          [ii]     $-\infty$          [iii]     $0$          [iv]     $1$          [v]     $-1$

[h]     For the exponential curve $f(x) = \left(\dfrac{5}{7}\right)^x$, as the value of $x \to -\infty$, the value of $y \to$
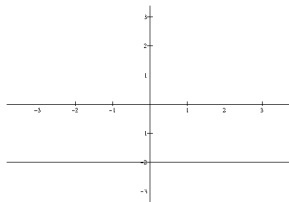        [i]     $\infty$          [ii]     $-\infty$          [iii]     $0$          [iv]     $1$          [v]     $-1$

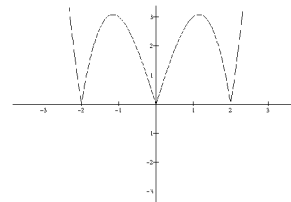[10]    Which of the following graphs represent one-to-one functions ?
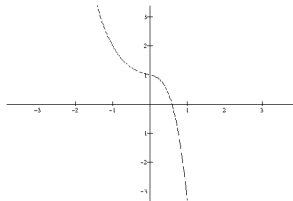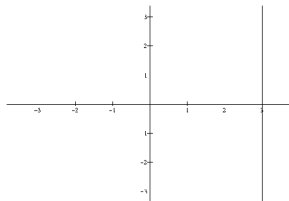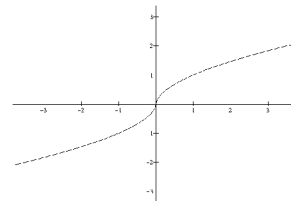
[a]
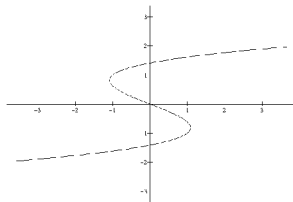
[b]
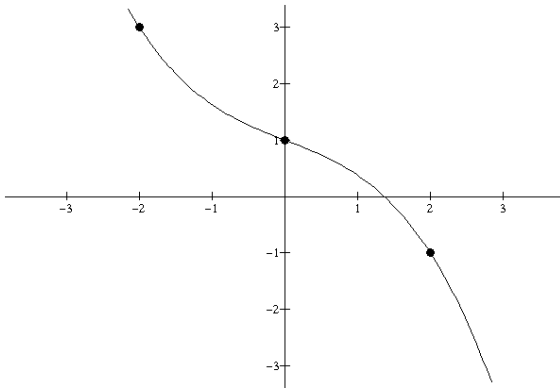
[c]

[d]

[e]

[f]

[11]    Find the inverses of the following functions.

[a]     $f(x) = \dfrac{9}{2} - \dfrac{3}{4}x$
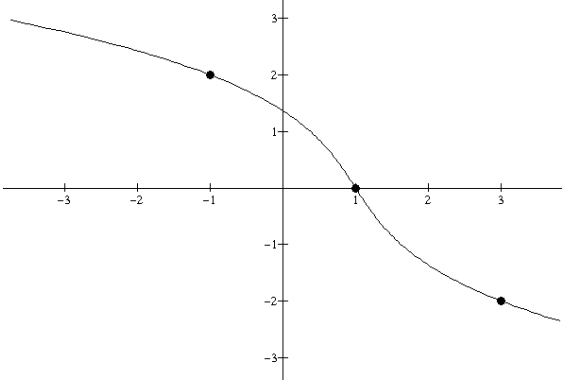
[b]     $f(x) = 4 - \sqrt{3 + 2x}$

[12]     Sketch the graph of the inverse of the following function.



## You may use a non-graphing calculator for the following

[13]     **Without using your calculator**, find the exact solution of the equation $6^{x-2} = 4^{x+1}$.
         Then, use your calculator to convert your exact solution into a decimal answer, rounded to 4 decimal places.
         **Check your answer.**

[14]     The number of bacteria in a colony is given by $B(t) = 1.3(2.1)^t$.

         [a]     How many bacteria were there at time $t = 4$ ? Round your answer to 1 decimal place.
         [b]     At what time were there at least 41 bacteria ? Round your answer to 2 decimal places.

[15]     [a]     Find the intensity (in microns) of an earthquake with a Richter magnitude of 5.6.
         [b]     Find the Richter magnitude of an earthquake of intensity 56,000,000 microns.

[16]     [a]     You take out a loan for $21,000 at 5.35% interest compounded monthly, and you make no payments on it.
                 how much do you owe 3 years later ?
         [b]     You take out a loan for $21,000 at 5.35% interest compounded weekly, and you make no payments on it.
                 How many years later will the total amount you owe be $30,000 ? Round your answer to 2 decimal places.
         [c]     How much should you deposit into an account that grows 5.35% compounded quarterly,
                 if you want the value of the account 4 years later to be $30,000 ?
         [b]     You take out a loan for $21,000 with interest compounded every $4$ months.
                 You make no payments on it, and 5 years later, you owe a total of $30,000.
                 What is the annual interest rate on the account ? Round your answer to 2 decimal places.

[17]     Draw the graph of $f(x) = -3 \cdot 2^{-x-1}$ using the process in the handout on my website. **LABEL ALL ASYMPTOTES CLEARLY.**

[18]     Draw the graph of $f(x) = 2\log_2\left(\dfrac{x+3}{2}\right)$ using the process in the handout on my website.

         **LABEL ALL ASYMPTOTES CLEARLY.**

# <u>ANSWERS</u>

[0]    [a]    for each value of input $x$, there is at most one value of output $y$
       [b]    for each value of input $x$, there is at most one value of output $y$
              and for each value of output $y$, there is at most one value of input $x$
       [c]    the set of all inputs that have a corresponding output
       [d]    the set of all outputs that have a corresponding input

[1]    [a]    all real numbers          [b]    $\{x > 0\}$          [c]    $\{x < 2\}$

[2]    [a]    $\{y > 0\}$          [b]    all real numbers

[3]    [a]    4          [b]    2          [c]    3          [d]    UNDEFINED
       [e]    UNDEFINED          [f]    0          [g]    6          [h]    4
       [i]    6          [j]    $-3$          [k]    7          [l]    UNDEFINED
       [m]    UNDEFINED          [n]    5

[4]    [a]    $-2$          [b]    3          [c]    11          [d]    $-5$
       [e]    5          [f]    7

[5]    [a]    $\log 24$          [b]    $\log 4$          [c]    $\log 32$          [d]    $\log x^3 y^2$

       [e]    $\log \dfrac{x^2 z}{y}$          [f]    $\log \dfrac{z}{y^2 x}$          [g]    $\log \dfrac{y^2 z^3}{x}$          [h]    $\log z^4 xy^3$

[6]    [a]    $\log 7 + \log 11$          [b]    $\log 13 - \log 5$          [c]    $8 \log 3$          [d]    $4 \log r + \log s$
       [e]    $5 \log a - 2 \log b$          [f]    $\log m - 2 \log n - 3 \log p$

       [g]    $2 \log x - \dfrac{1}{2} \log y - \dfrac{1}{2} \log z$

[7]    [a]    all real numbers          [b]    $\left\{ x \neq \dfrac{3}{2} \right\}$          [c]    $\{x \leq 8\}$

[8]    [a]    $\{y \neq -4\}$          [b]    $\{y \leq 7\}$

[9]    [a]-[ii]                    [b]-[v]                    [c]-[i]                    [d]-[ii]
       [e]-[i]                     [f]-[iii]                  [g]-[iii]                  [h]-[i]

[10]   [a]    no     [b]    no     [c]    no     [d]    yes     [e]    no     [f]    yes

[11]   [a]    $f^{-1}(x) = 6 - \dfrac{4}{3} x$          [b]    $f^{-1}(x) = \dfrac{(4-x)^2 - 3}{2}$

[12]



[13]   $\dfrac{2 \log 6 + \log 4}{\log 6 - \log 4} \approx 12.2571$

[14]   [a]    25.3 bacteria          [b]    4.65 units of time
[15]   [a]    398107 microns          [b]    7.748
[16]   [a]    $24647.26          [b]    6.67 years          [c]    $24254.83          [d]    7.22%
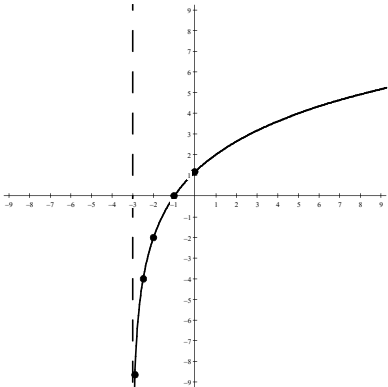
[17]

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $f(x)$ | $-12$ | $-6$ | $-3$ | $-1.5$ | $-0.75$ |
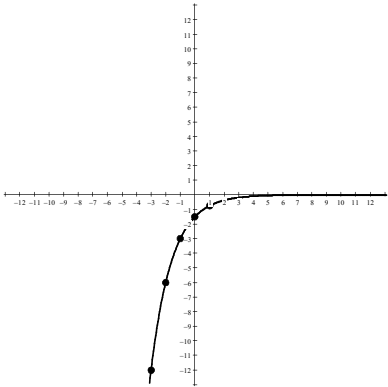


horizontal asymptote at $y = 0$

[18]

| $x$ | $-2.9$ | $-2.5$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|---|---|
| $f(x)$ | $-8.6$ | $-4$ | $-2$ | $0$ | $1.2$ |



vertical        asymptote at $x = -3$